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RESEARCH ARTICLE OPEN ACCESS

An Optimal Linearized Quali-Quantitative Multi-Objective Simulation Model to Planning and Managing Integrated Surface Resources Systems

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ABSTRACT

This paper presents a multiobjective mathematical model that simulates integrated quali-quantitative aspects of water, using linear programming techniques, which may be used to evaluate the sustainability of existing or planned water resources scenarios in a watershed. One of its main features, which differentiates it from other simulation models available in the literature, is the objective function that incorporates weighted meeting requirements of multiuse water quantity demands, operational targets and the meeting of water quality parameters goals, the last one in accordance with the standards of the Brazilian CONAMA's law. The non-linear mathematical description of hydraulic, water quality and operational processes for water demands, rivers and reservoirs, which are constraints of the optimization model, were appropriately linearized. Biochemical oxygen demand, dissolved oxygen, total phosphorous, total nitrogen, chlorophyll-a and fecal coliforms water quality parameters were included in the model. Some indicators of performance analysis, such as reliability, vulnerability, resilience and sustainability were also included.

***Keywords* -** Water resources, Simulation, Linear programming, Quali-quantitative approach

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I. INTRODUCTION

Water shortage is a chronic problem that affects the entire planet. Its aggravation is related to the economic and population growth that requires a significant increase of water demands or cause pollution of the water bodies. Water quantity and quality crisis is already a today's reality in most regions around the world and requires an increasing phenomena and variables representation complexity to planning and managing water bodies towards reaching the water system sustainability. Under these conditions the evaluation of the water related problems can no longer be restricted to a simple water balance between supply and demand or a simple estimation of its pollution, but should also consider their interrelationships and meet the uses, the geo-environmental and socio-cultural peculiarities or requirements to achieve and ensure a certain level of a region quality of life. Therefore, the better the systemic and holistic mathematical model conception to be used, which takes into account the multi-objectives of the closest representation of water quantity and quality multiple uses requirements, operation and hydrology dynamics, the better the analysis and managing solutions to be provided.

In the search for a solution to the complex problems of water resources planning, water resources managers have used techniques and tools, based on mathematical and computational approaches, including techniques of simulation and optimization, to assist in the operation, processes' analysis, planning and decision-making in water resources system. However, is no longer acceptable the use, only, classic simulation models or optimization with only one goal, as, for example, the maximization of economic efficiency. Following the new trends in the treatment of water resources problems, the inclusion of quantifiable more generic objectives, allowing the consideration of economic, social, political, environmental and other aspects has become necessary (Labadie, 2004 & Wurbs, 2005).

The current trend on water resources mathematical modeling is to match the use of simulation and optimization techniques. For Simonovic (1992), this approach helps to reduce or eliminate the gap between practice and theory in water resources system analysis. For Wurbs (2005), various solutions and strategies can be obtained with the use of combined techniques of simulation and optimization. Several successful examples with combined use of these techniques were employed in solving problem of water resources planning and management.

For Latorre et al. (2014), the planning of water use of a watershed requires essentially two approaches: optimization and simulation. Both techniques are complementary, i.e. once used a given optimization model, its results could be used to detail a long-term planning and analyzed by a simulation model. The main features in the modeling of multi-reservoirs systems can be summarized as: to be based on simulation and optimization techniques; focus on the account of resources (water, energy, costs, between others.) allocation and consumption and hydrological processes; to use a systems approach with different levels of detail focusing on connected and unconnected rivers and reservoirs and the non-linearity being explicitly considered or approximate.

For Huang (2014) water quality models are usually developed to simulate the fate and transport of contaminants in rivers, reservoirs and lakes. They are useful tools in the management of water resources and may contain parameters that cannot directly be measured or are measurement time-consuming, but can be estimated and adjusted to better represent the reality of the system.

To address the problems of water resources, it is no longer enough to independently consider the issues of quantity and quality of water, they should be considered in an integrated manner, even in the objective function. Ray et al. (2010) demonstrated this tendency and applied an integrated multiobjective linear deterministic model to minimize the cost of water supply, wastewater disposal and maximize the re-use of water in the Lebanon capital as a future option for semiarid regions. They applied linear and non-linear techniques, being the last one structured by a linearization process.

According to Han et al. (2013) the conception of a mathematical modeling still persists for water allocation and to define strategies for large-scale river basin systems, considering social, environmental and economic aspects to obtain, as a result, an integrated framework for sustainable water allocation.

Meanwhile other studies use approaches of multiples models for to integrate and model the components of water system. As Lee et al (2021) that maximized sustainability index of a water distribution system; Hu et al (2020) analyzed real-time coupling of hydro-environmental model and scalar transport Model and observed that both time-consuming computational; Zhao and Cai (2020) evaluated reservoir operation rules using HM-DT model and demonstrated that they can derive a limited number of operation rules between different changing conditions.

Although the majority of the models, in particular those of simulations, are quite versatile and spread, they cannot yet incorporate accurately features such as the nonlinearities of hydraulic and operational processes as well as the multiobjective features when considering qualitative (concentration) and quantitative (allocation) aspects of water resources.

Therefore, this paper presents a monthly river basin water resources simulation model that takes into account, in an integrated way, water quantity and quality variables. Within each month it is generated and solved a linear optimization model, which is solved via sequential linear programming to couple with the model's nonlinearities. The weighted multiple objective function, which establishes their fulfilling priorities, is related to meeting multiple uses water quantity requirements, reservoirs target volumes, and water quality parameter standards (concentrations of biochemical oxygen demand, dissolved oxygen, total phosphorus, total nitrogen, fecal coliforms and chlorophyll-a) as well as minimizing the reservoir spills. The model's nonlinearities are related to the mathematical description of hydraulic processes and mass balance for each water quality parameter.

Seeking to evidence scientific research contribution and innovation to the theme "Simulations Models" it was used the combined techniques and linearization devices. The use tools aimed to provide an Optimal Quali-Quantitative Multi-Objective Simulation Model fully linearized and with computational time of fews minutes for great water systems.

II. SIMULATION MODEL AND ITS PECULIARITIES

The world trend is to develop mathematical models for the planning of water systems, using combined techniques of simulation and multiobjective optimization and including not only the quantitative aspect as well as the qualitative aspect of water resources in an integrated manner. Several of the models seen in literature are related with water allocation and water quality. In general, they provide results for water allocation and, in the sequence, determine the concentrations of different water quality parameters in control points in rivers and reservoir, that is, the quantitative and qualitative water resources analysis are not performed simultaneously.

The ACQANET (Azevedo et al., 1998), MODSIM (Labadie, 2004) and MIKE BASIN (DHI, 2001) are examples of simulation models that use different modules of separated analyses and the solver is based on network flows techniques, in special the one based on the algorithm "Out-of-Kilter". Although the algorithm allows the structuring of networks with a large number of reservoirs, demands and channels (links), on the other hand imposes limitations on water resources systems modelling, mainly the ones related to sequential programming to couple with the system's nonlinearities. There are several applications of these network flows models, such as the work of Berhe et al (2013) that used the MODSIM model to analyze water demand allocations through the set up of four scenarios for the Awash River basin of Ethiopia in Africa. Other research developed by Kaiglová and Langhamme (2013) applied the model to analyze, through MIKE BASIN different scenarios, the pollution levels of the Olšava River basin situate between the Central Europe and the Czech and Slovakia Republic. There are other models in the literature using the techniques of network flows model such as WATERWARE, RIVERWARE, HEC-PRM, HEC-ResSim, among others.

Roberto (2002) recognize that, although the network flows models present computational efficiency, they also present some operational and restrictive limitations to the representation of water resources system. Some of them are: these types of algorithms only optimize linear systems (they do not consider the nonlinearities of the system); the objective function is pre-set and therefore not introduce other user goals and the two types of constraints used are the conservation of mass in control nodes and the limitation of the flow in the arches. The solution, usually, is achieved through the determination of the variables and their analysis done at each time interval, i.e. does not guarantee the global optimum for a period of n intervals of time.

For Vieira et al. (2012) does not exist a general methodology that takes into account all possible configurations and requirements of a water resources system, targeted to analyze the performance of a network of reservoirs and rivers at the same time and space. The chosen methodology to describe and be applied to water resources system depends on: their quantitative and qualitative limitations, system interface integration, the availability and use of water resources, availability of data and computational efficiency. Based on this, although there still exists gaps to overcome between theory and practice, computational and mathematical modeling have advanced a lot over the last years.

III. MATERIALS AND METHODS

The proposed simulation model for surface water systems, despite performing mostly as nonlinear phenomena, was developed using linear programming techniques. These techniques allow one to have a great deal of flexibility in modeling, low processing time, convergence to an global optimal solution and are widely used in solving large problems of water systems. Many of the nonlinearities of the problem was incorporated through processes of linearization and search for solutions was made via iterative processes. The interior point method of MATLAB (Matrix Laboratory) optimization library package, version 6.5, was used to search for the optimal solution.

That simulation model works on a monthly based time scale (Figure 1) and has, in its core, an optimization algorithm, which uses linearization techniques such as sequential linear programming and linear approximations programming and are applied at each time step. From the information obtained for the variables at time t-1, an optimal multiuse water allocation is determined for the time t, while considering hydro climatic conditions (precipitation, evaporation, water storage, etc.), hydraulic components (reservoirs, irrigation systems, etc.), water quantity and quality demand requirements, water and mass balances (applied to reservoirs, river nodes/control points and agricultural systems), between others.

Other than that fixed water demands are related to urban and rural water supply, fish farming, ecological and electric power generation demands, etc.. The irrigation water demand is determined according to climatic cultures supplementary water requirements, attained from soil water balance (Curi et. al, 2005). It has rational model return flow, proposed by Von Sperling (1995) was used to estimate the average effluent return flow from the cities and irrigated perimeters. Despite being a relatively coarse approach to represent small watersheds, they can be applied to the studied watershed due to its low contribution to this flow (Villela and Mattos, 1975).

The water quality parameters of the model are the biochemical oxygen demand, dissolved oxygen, total nitrogen, total phosphorus, chlorophyll-a and fecal coliform and their related demand concentrations goals are regulated by a Brazilian law named CONAMA 357/05 in terms of classes of water bodies quality standards. The water quality processes are related to pollution discharge of sewers and irrigation drainage, as well as their transportation and riverbed and reservoir systems self-depuration.

A system approach was used to represent any surface water system (Figure 2). The system is composed by components (reservoirs, irrigated perimeters, water demands, inflows, and outflows), links (river and channels) and nodes (interconnection points of components and links as well as water quality control points).

The multi-objective function can integrate, at the same time, both the qualitative and the quantitative operational aspects. The objectives of the objective function are associated to weighting factors. The weights represents the priority levels of the fulfillment of the function objectives. Therefore, given the characteristics of the problem's objectives, it is required the normalization of the objectives. The priority levels are represented by positive integer numbers where the higher the number the most preferred one to be fulfilled.

Furthermore execution of the proposed model starts with the quantitative simulation, to determine an optimum monthly water volume allocation. The quantitative simulation variables solution serve as initial point to the quali-quantitative simulation model, which is solved via sequential linear programming. The solution for the quali-quantitative water variables is reached when constraints are satisfied and stopping criteria is reached.

The physical components of a water resources system are represented by icons, as shown on Figure 2, having each one its own meaning, physical parameters and variables.

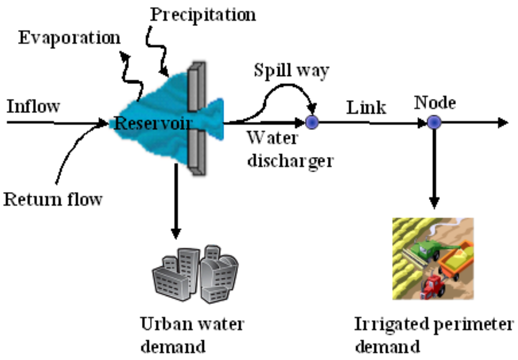

Fig. 2. General representation of surface water system using model's components

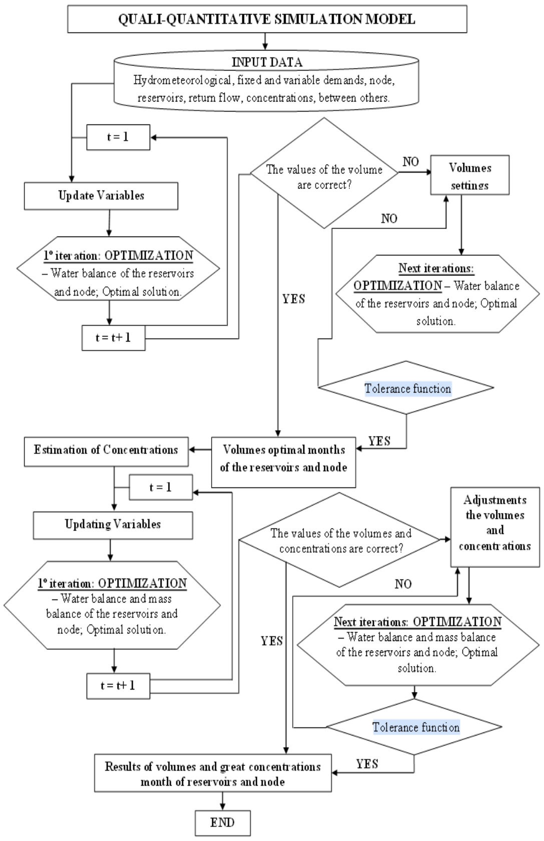

Fig. 1. Quali-quantitative simulation model flowchart.

IV. RESULTS AND DISCUSSION

4.1 Multi-objective function

The multiple objectives to be optimized in a given water system are established by a mathematical function, which takes into account some preferences of different water users and decision makers. The model objective function covers both quantitative aspects, characterized by maximizing the satisfaction of the fulfillment of the multiple uses' water demand (flow rates and volumes) requirements and other their operation aspects, as well as qualitative aspects that represent the meeting of water quality parameter concentrations goals required at the river basin control points. Five main objectives of the objective function [$FO_i(t)$, i = 1, .., 5, over the time t = 1, ..., n] are taking into account, and are represented by:

$$Z = \sum_t [F01_t + F02_t + F03_t + F04_t + F05_t] \qquad (1)$$

The first objective function, $F01_t$, is to maximize the fulfillment of multiple users' water demand requirements and is represented by:
Maximize

$$FO1_t = \sum_{d(r)} \alpha_{1,d(r)} * Ra_{d(r),t} + \sum_{d(r)} \alpha_{2,d(r)} * Rip_{d(r),t} + \dots$$
$$+ \sum_{d(r)} \alpha_{3,d(r)} * Ris_{d(r),t} + \sum_{d(r)} \alpha_{4,d(r)} * Re_{d(r),t} \qquad (2)$$

where α* is the weight coefficient, which establishes the objectives priorities, $Ra_{d(r),t}$ is the d-th allocated human water supply volume from reservoir r at month t; $Rip_{d(r),t}$ is d-th irrigation allocated water volume to perennial crops from reservoir r at month t; $Ris_{d(r),t}$ is d-th irrigation allocated water volume to seasonal crops from reservoir r at month t; and $Re_{d(r),t}$ is the d-th ecological water volume released by reservoir r at month t.

Already objective function F02 is related to the meeting of operational reservoirs goals and is represented by:

Maximize

$$FO2_t = \sum_{r,t} \alpha_{5,r} * SC_{r,t} + \sum_{r,t} \alpha_{6,r} * SM_{r,t} \qquad (3)$$

where $SC_{r,t}$ is the minimum r-th reservoir volume at month t and $SM_{r,t}$ is the target r-th reservoir volume at month t.

The objective function F03 is related to water losses by reservoirs spillage and is represented by:

Minimize

$$FO3_t = \sum_{r,t} \alpha_{6,r} * Sp_{r,t} \qquad (4)$$

where $SP_{r,t}$ is the spilled water volume from reservoir r at month t.

The objective function F04 aims to minimize water quality pollution concentration levels regarding to meeting goals for biochemical oxygen demand (BOD), total nitrogen (NT), total phosphorus (FT) and fecal coliforms (CF) for each month t, and is represented by:

Minimize

$$FO4_t = \sum_{r,t} \alpha_{7,r} * CBOD_{r,t} + \sum_{r,t} \alpha_{8,r} * CNT_{r,t} + \dots$$
$$+ \sum_{r,t} \alpha_{7,r} * CFT_{r,t} + \sum_{r,t} \alpha_{8,r} * CCF_{r,t} + \dots$$
$$+ \sum_{c,t} \alpha_{9,c} * CBOD_{c,t} + \sum_{c,t} \alpha_{10,c} * CNT_{c,t} + \dots \qquad (5)$$
$$+ \sum_{c,t} \alpha_{11,c} * CFT_{c,t} + \sum_{c,t} \alpha_{12,c} * CCF_{c,t}$$

where $CBOD_{r,t}$ is the BOD concentration level for waters of reservoir r at month t; $CNT_{r,t}$ is the NT concentration level for waters of reservoir r at month t; $CFT_{r,t}$ is the concentration level of FT for waters of reservoir r at month t; $CCF_{r,t}$ is the concentration level of CF for waters of reservoir r at month t; $CBOD_{c,t}$ is the BOD concentration level in node c at month t; $CNT_{c,t}$ is the NT concentration level in control point c at month t; $CFT_{c,t}$ is the FT concentration level in control point c at month t and $CCF_{c,t}$ is the CF concentration level in control point c at month t.

The objective function F05 aims at maximizing the dissolved oxygen (OD) in controls points along the river basin and in reservoirs:

Maximize

$$FO5_t = \sum_{r,t} \alpha_{13,r} * COD_{r,t} + \sum_{c,t} \alpha_{14,c} * COD_{c,t} \qquad (6)$$

where $COD_{r,t}$ is the OD concentration level in reservoir r at month t; $COD_{c,t}$ is the OD concentration level in control point c at month t.

Equality constraints

Reservoirs

That is water balance equation used to determine the monthly the stored volumes in reservoirs:

$$S_{r,t} = S_{r,(t-1)} + I_{r,t} + Sd_{c,t} - \sum_{d(r)} R_{d(r),t} - \dots$$
$$- Qf_{r,t} - Sp_{r,t} + \left[P_{r,t} - E_{r,t}\right] * Am_{r,t} \qquad (7)$$

where $S_{r,t}$ is the stored water volume in reservoir r at month t; $S_{r,(t-1)}$ is the stored water volume in reservoir r at month t-1; $R_{d(r),t}$ is the d-th allocated water volume from reservoir r at month t; $Qf_{r,t}$ is the discharged water volume from reservoir r at month t; $Sp_{r,t}$ is the spilled water volume from reservoir r at month t; $I_{r,t}$ is the inflow water volume in reservoir r at month t; $P_{r,t}$ is the precipitation over reservoir r at month t; $E_{r,t}$ is the evaporation of reservoir r at month t; $Am_{r,t}$ is the average water surface area of reservoir r at month t; $Sd_{c,t}$ is the water inflow from the c-th upstream node of reservoir r at month t.

The reservoir average surface area is determined by:

$$Am_{r,t} = [\frac{A_{r,t} + A_{r,(t-1)}}{2}] \qquad (8)$$

where $A_{r,t}$ is the r-th reservoir water surface area, in m², at month t and $A_{r,(t-1)}$ is the r-th reservoir water surface area, in m², at month t-1.

To calculates the reservoir water surface area (AS) of the r-th reservoir at time t established a relationship with the reservoir stored water volume (VA) according to a piecewise linear function as shown on Figure 3.

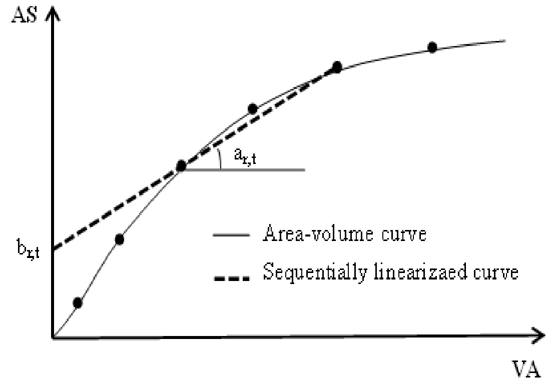

Fig. 3. Piecewise linearization of a reservoir’s water stored volume and its water surface area

As the function describing the reservoir surface area in terms of reservoir storage volume is nonlinear, it has been replaced by a linear approximation and redefined for a determined reservoir volume at each iteration when applying the sequential linear programming. Therefore, equation 9 is replaced by:

$$Am_{r,t} = \left[\frac{a_{r,t} * S_{r,t} + b_{r,t} + a_{r(,t-1)} * S_{r(,t-1)} + b_{r(,t-1)}}{2}\right] \quad (9)$$

where $a_{r,t}$ is the slope of the function relating area and volume of reservoir r at month t; $a_{r,(t-1)}$ is the slope of the function relating area and volume of reservoir r at month t-1; $b_{r,t}$ is the corresponding linear function intercept for reservoir r at month t; $b_{r,(t-1)}$ is the corresponding linear function intercept for reservoir r at month t-1.

To meet some reservoir operational requirements, such as the satisfaction of electric power plant, fish farm or leisure requirements, it is introduced the concept of the r-th reservoir target volume, $Smeta_r$, which is defined by:

$$S_{r,t} = Smeta_r + SMP_{r,t} - SMN_{r,t} \quad (10)$$

where $SMP_{r,t}$ is the reservoir useful volume stored above the r-th reservoir target volume in month t and $SMN_{r,t}$ is the reservoir volume required to reach the r-th reservoir target volume in month t and should be maximized.

A problem that typically occurs in semi-arid regions is the high evaporation rate, which may cause the reservoir reach its minimum stored volume, $Smin_r$, and quickly may become completely empty. To account for that, the following equation is used:

$$S_{r,t} = Smin_r + SCP_{r,t} - SCN_{r,t} \quad (11)$$

where $SCP_{r,t}$ is the volume useful stored above the r-th reservoir minimum storage volume in month t; $SCN_{r,t}$ is the required volume to reach the r-th reservoir minimum storage volume in month t.

One of the main goal of this modeling is to estimate average values for concentrations that reflect the reservoir's water quality, which are indispensable in an integrated water system planning. Herein, the reservoir water quality parameters are considered constants during the month t, which means that it is considered a water total mix. Moreover, their volumes are regarded to be constant within months t and t-1 (TUCCI, 2005). To minimize the time effects on the qualitative analysis of reservoir volumes, an average of evaporated and precipitated volumes is determined.

The monthly mass balance equation in a reservoir, which is not in series, for a given parameter P, such as BOD, FT, NT or CF, is calculated by the following general equation:

$$CP_{r,t} = \left[\frac{I_{r,t} * CI_{r,P,t} + \sum Qr_{r(a),t} * Cr_{r(a),P,t}}{I_{r,t} + \sum Qr_{r(a),t} + K_{1,r,P} * S_{r,t}}\right] * \ldots$$
$$* \left[1 - e^{-\left(\frac{I_{r,t} + \sum Qr_{r(a),t}}{S_{r,t}} + K_{1,r,P}\right)}\right] + \ldots \quad (12)$$
$$+ CP_{r(,t-1)} * e^{-\left(\frac{I_{r,t} + \sum Qr_{r(a),t}}{S_{r,t}} + K_{1,r,P}\right)}$$

The OD water quality parameter is determined by:

$$COD_{r,t} = \left[\frac{\begin{array}{l} I_{r,t} * CI_{r,OD,t} + \ldots \\ \sum Qr_{r(a),t} * Cr_{r(a),OD,t} + \ldots \\ + K_{2,r,OD} * CS_{r,OD} * S_{r,t} - \ldots \\ - K_{1,r,BOD} * KT_{r,t} * C_{r,BOD,t} * S_{r,t} \end{array}}{I_{r,t} + \sum Qr_{r(a),t} + K_{2,r,OD} * S_{r,t}}\right] * \ldots$$
$$* \left[1 - e^{-\left(\frac{I_{r,t} + \sum Qr_{r(a),t}}{S_{r,t}} + K_{2,r,OD}\right)}\right] + COD_{r(,t-1)} * \ldots \quad (13)$$
$$* e^{-\left(\frac{I_{r,t} + \sum Qr_{r(a),t}}{S_{r,t}} + K_{2,r,OD}\right)}$$

where $K_{1,r,BOD}$ is the BOD decay coefficient for reservoir r; $CP_{r,t}$ is the concentration of a water quality parameter P in reservoir r at month t; $CP_{r,(t-1)}$ is the concentration of a water quality parameter in reservoir r at month t-1; $K_{2,r,OD}$ is the OD coefficient for reservoir r; $COD_{r,t}$ is the OD concentration in reservoir r at month t; $C_{r,BOD,t}$ is the BOD concentration in reservoir r at month t; $COD_{r,(t-1)}$ is the OD concentration in reservoir r at month t-1; $Qr_{r(a),t}$ is the a-th inflow water volume in reservoir r at month t; $Cr_{r(a),P,t}$ is the a-th concentration of parameter P of the inflow $Qr_{r(a),t}$ in reservoir r at time t; $KT_{r,t}$ is a constant to transform 5 days BOD data into monthly BOD in reservoir r at month t; $CI_{r,P,t}$ is the inflow ($I_{r,t}$) concentration of water quality parameter P in reservoir r at month t. In case of having reservoirs in series, the mass balance equation also accounts for the $Sd_{c,t}$ inflow volume from c-th upstream node of reservoir r at month t with its related $CadP_{c,t}$ concentration of water quality parameter P.

A linear approximation method was used to linearize each water quality parameter mass balance equation, which is a non-linear process, to make it possible to integrate water balance and mass balance calculations in an optimization routine that is triggered at each monthly calculation.

River

To analyze the water flow in a river bed or creek along its length, nodes or control points are used, which allow a connection of two or more components (reservoir, links, between others.) and where is performed the water and mass (water quality) balance. Besides being points of connection among water components, they can be used in different situations such as to determine the concentration of water quality parameters anywhere in the water system. The following equation represents the water balance in a node downstream the reservoir r:

$$Qf_{r,t} + Sp_{r,t} + Re_{r,t} = SR_{r,c,t} \tag{14}$$

where $SR_{r,c,t}$ is the water discharged by reservoir r at month t reaching node c.

For a generic node, the mass balance equation is given by:

$$SR_{r,c,t} + Qa_{c,t} = Sd_{c,t} \tag{15}$$

where $Qa_{c,t}$ is related to other water sources volume inflow in node c at month t.

The mass balance for a water quality parameter P (Biochemical Oxygen Demand, Dissolved Oxygen, Total Nitrogen, Total Phosphorous or fecal coliform) is given by:

$$\begin{aligned} &SR_{r,c,t} * CadP_{r,c,t} + \sum_{c(a)} Qr_{c(a),t} * Crc_{c(a),t} + \ldots \\ &+ Qa_{c,t} * Ca_{c,t} = Sd_{c,t} * CndP_{c,t} \end{aligned} \tag{16}$$

where $CadP_{r,c,t}$ is the concentration of water quality parameter P that reaches node c from reservoir r at month t; $Qr_{c(a),t}$ is the a-th water user return volume that reaches node c at month t; $Crc_{c(a),t}$ is the concentration of the P water quality parameter associated with $Qr_{c(a),t}$; $Ca_{c,t}$ is the concentration of the P water quality parameter associated with $Qa_{c,t}$; $CndP_{c,t}$ is the concentration of the P water quality parameter associated with the outflow water volume $Sd_{c,t}$ from noce c at month t.

The autodepuration equation for a water quality parameter P (DBO, NT, FT or CF), according to the model of Streeter-Phelps (1925), apud Tucci (2005), is given by:

$$CadP_{r,c,t} = CP_{r,t} * exp\left(-k1 * \frac{Li * Bmi * Pmi}{Sd_{r,c,t}} \right) \tag{17}$$

where L_i is i-th the river length; $k1_i$ is the i-th river deoxygenation rate; B_{mi} is the river cross-section length; P_{mi} is the i-th river's average depth.

If the water quality parameter is the dissolved oxygen (OD), then the following Streeter-Phelps equation (Tucci, 2005) is used:

$$CadOD_{r,c,t} = CS - \left[\begin{array}{l} \dfrac{k1 * L_{r,t}}{k2 - k1} * \ldots \\ * \left\langle \begin{array}{l} exp\left(-\dfrac{k1 * Li * Bmi * Pmi}{SRr,\ c,t} \right) - \ldots \\ - exp\left(-\dfrac{k2 * Li * Bmi * Pmi}{SRr,\ c,t} \right) \end{array} \right\rangle + \ldots \\ + \left(CS_t - COD_{r,t}) \right) * \ldots \\ * exp\left(-\dfrac{k2 * Li * Bmi * Pmi}{SRr,\ c,t} \right) \end{array} \right] \tag{18}$$

where $CadOD_{r,c,t}$ is the OD concentration of the water flow that reaches node c from reservoir r at month t; $L_{r,t}$ is DBO concentration of the water flow that leaves reservoir r and reaches node c at month t; CS_t is the local saturation concentration at month t.

To achieve the water quality parameters concentration CONAMA's goals in control node c, as well in reservoirs, the post contractual method (Lanna, 1998) was used as follows:

$$CndP_{c,t} = CPmeta_{c,t} + CPP_{c,t} - CPN_{c,t} \tag{19}$$

where $CPN_{c,t}$ is the concentration of the water quality parameter P, which is below the targeted one, in node c at month t; $CPP_{c,t}$ is the concentration of the water quality parameter P, which is above the targeted one, in node c at month t and $CPmeta_{c,t}$ is the targeted concentration of the water quality parameter P in node c at month t.

High concentration of phosphorous and nitrogen in water bodies may contribute to the increase of algae and get it into an eutrophication process. As the chlorophyll water quality parameter, CLA, indicates the presence of algae, but is not usually measured, Lamparelli (2004) have proposed a mathematical relationship to determine the chlorophyll CLA (µg/l) concentration levels in rivers, which may also be used in reservoirs, in function of FT (μg/l), as follows:

$$CCLA_{c,t} = 0{,}081 * \left(CndFT_{c,t}\right)^{1{,}24} \tag{20}$$

where $CCLA_{c,t}$ is the CLA concentration at node c at month t; and $CndFT_{c,t}$ is the concentration of FT at node c at month t.

Inequality constraints

Reservoirs

The monthly operational inequality constraints associated with reservoirs regarding its decision variables, $S_{r,t}$, $Rd_{(r),t}$, $Qf_{r,t}$ e $Sp_{r,t}$, which has been used herein, are:

$$0 \leq R_{d(r),t} \leq Dmax_{d(r),t} \tag{21}$$

$$0 \leq S_{r,t} \leq Smax_{r,t} \tag{22}$$

$$0 \leq Qf_{r,t} \leq Qfmax_{r,t} \tag{23}$$

$$0 \leq Sp_{r,t} \leq Spmax_{r,t} \tag{24}$$

where $Dmax_{d(r),t}$ is the maximum volume of water intake from reservoir r to meet the d-th water demand at month t; $Qfmax_{r,t}$ is the maximum allowed discharge of reservoir r at month t; $Spmax_{r,t}$ is the maximum water spillage from reservoir r at month t; $Qfmax_{r,t}$ is determined (QUINTELA, 1981) by:

$$Qfmax_{r,t} = Cf_r * Af_r * (H_{r,t} - Hg_r)^{0{,}5} \tag{25}$$

where Cf_r is the discharge coefficient of reservoir r; Af_r is the cross-sectional area of the r-th reservoir discharger; Hg_r is the quota of the bottom of the discharger orifice at reservoir r; $H_{r,t}$ is the quota of the water level in reservoir r at month t.

With help equacion which determines the maximum reservoir water discharge $Qfmax_{r,t}$, a piecewise linear relationship between the volume of the reservoir and its maximum water discharge, as shown in Figure 4, is created and used in the sequential linear programming approach.

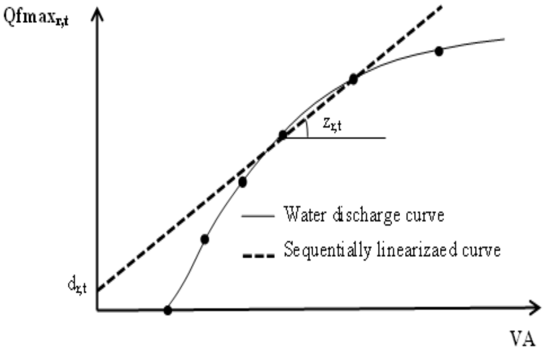

Fig. 4. Piecewise linearization of the r-th maximum reservoir water discharge.

Therefore, the linear constraint for the reservoir's water discharge representation is given by:

$$0 \leq Qf_{r,t} \leq z_{r,t} * S_{r,t} + d_{r,t} \tag{26}$$

where $z_{r,t}$ is the slope of the line determined for the r-th reservoir discharge at month t; $d_{r,t}$ is the constant term of the line determined for the r-th reservoir discharge at month t.

The maximum overflow reservoir spillage is determined by (QUINTELA, 1981):

$$Spmax_{r,t} = Cv_r * Bv_r * (Hvmax_{r,t} - Hsol_r))^{1{,}5} \tag{27}$$

where Cv_r is the discharge coefficient of the r-th reservoir spillway; Bv_r is the r-th reservoir spillway effective base length; $Hvmax_{r,t}$ is the quota of the r-th reservoir water level at maximum allowed spillage; $Hsol_r$ is the quota of the r-th reservoir spillage base.

Reservoir volumes constraints, regarding to their targets, minimum and maximum volumes, were also included and represented by:

$$0 \leq SMN_{r,t} \leq Smeta_{r,t} \tag{28}$$

$$0 \leq SMP_{r,t} \leq Smax_{r,t} - Smeta_{r,t} \tag{29}$$

$$0 \leq SCN_{r,t} \leq Smin_{r,t} \tag{30}$$

$$0 \leq SCP_{r,t} \leq Smax_{r,t} - Smin_{r,t} \tag{31}$$

where $Smax_{r,t}$ is the maximum r-th reservoir water volume at month t.

Others components constraints

Variables regarding to inflows and outflows at nodes may be also limited by a maximum value, as follows:

$$0 \leq SR_{r,c,t}(t) \leq DSmax_{r,c,t} \tag{32}$$

$$0 \leq Sd_{c,t}(t) \leq Sdmax_{c,t} \tag{33}$$

$$0 \leq Qa_{c,t} \leq Qamax_{c,t} \tag{34}$$

where $DSmax_{r,c,t}$, $Sdmax_{c,t}$ and $Qamax_{c,t}$ are the maximum allowed volumes for $SR_{r,c,t}$, $Sd_{c,t}$ and $Qa_{c,t}$, respectively in node c at month t.

The constraints regarding to reservoir water withdrawals to meet several water users' demands are represented by:

$$0 \leq Ra_{d(r),t} \leq Damax_{d(r),t} \tag{35}$$

$$0 \leq Rip_{d(r),t} \leq Dipmax_{d(r),t} \tag{36}$$

$$0 \leq Ris_{d(r),t}(t) \leq Dismax_{d(r),t} \tag{37}$$

$$0 \leq Re_{d(r),t}(t) \leq Demax_{d(r),t} \tag{38}$$

where $Damax_{d(r),t}$ is the maximum d-th user water withdraw from reservoir r at month t; $Dipmax_{d(r),t}$ is the maximum water withdraw from reservoir r for the d-th perennial crop water demand at month t; $Dismax_{d(r),t}$ is the maximum water withdraw from reservoir r for the d-th seasonal crop water demand at month t; $Demax_{d(r),t}$ is the maximum water withdraw from reservoir r for ecological needs at month t.

The water quality parameters may be constrained by upper and lower bounds as follows:

I – upper bounds for a P water quality parameter concentration of the inflow that reaches reservoir r or node c at month t:

$$0 \le CadP_{r,c,(t)} \le CadPmax_{r,c,t} \qquad (39)$$

II – upper bounds for a P water quality parameter concentration of the outflow that leaves node c at month t:

$$0 \le CndP_{c,t} \le CndPmax_{c,t} \qquad (40)$$

III – upper bound for an above deviation from the target value for the P water quality parameter concentration at node c or reservoir r at time t:

$$0 \le CP^{+}_{r,c,(t)} \le CndPmax_{r,c,t} - CPmeta_{r,c,t} \qquad (41)$$

System performance indicators

The reliability, resilience and vulnerability risks performance indexes (Hashimoto et al., 1982) are determined at each simulation, when comparing the systems requirements versus the response attained from the simulation model.

The reliability (*C*) is the percentage of time the system has been operating without failure and are represented by:

$$C = (1 - \frac{NF}{NT}) * 100\% \qquad (42)$$

where *NF* is number of failures; and *NT* is simulated period total.

Already the resilience (*R*) is the ability of a system to recover from failure and are calculated by the following equation.

$$R = (\frac{NR}{NPI}) * 100\% \qquad (43)$$

where *NR* is number of times the reservoir left an unsatisfactory state; and *NPI* is unsatisfactory number of periods.

In the case of vulnerability (*V*) is the magnitude of the failures to which the system is subjected and this way:

$$V = (\frac{NSD}{NF}) * 100\% \qquad (44)$$

where *NSD* is Deficit volume during a period of continuous failure; and *NF* is number of total failures sequentially.

Loucks (2000) proposed a general sustainability index (S) defined by the product between reliability, resilience and non-vulnerable part.

$$S = C * R * [1 - V] \qquad (45)$$

Model's convergence analysis

The presented simulation model, which is based on sequential linear programming due its intrinsic representation of the processes nonlinearities, has a convergence measure to its iterative process. The less the error imposed to attain a final result, the better its accuracy. The generally used value for the error limit (e) of the objective function at month t, FO_t with FO_{t-1}, function at month t-1, which is given by the following equation, using a tolerance (to) of is 0.0001.

$$e = \left| \frac{FO_t - FO_{(t-1)}}{FO_t} \right| \le to \qquad (46)$$

V. CONCLUSION

There are many water resources computational simulation models in the literature, based or not on linear programming. Whatever is the chosen model, there will be some mathematical simplification or constraint formulation that may or may not compromise the precision, requirement or adaptability of the model to the analysis of a studied system. These limitations arises from the considered number of hydraulic components, multiobjective function, linearization of hydro-climatic processes, capability of applied numerical programming techniques, and so on. The model proposed herein is an attempt to better fulfil modern practical water allocation requirements, depending upon the availability of required data, while using proven efficient standard worldwide known techniques.

The main contributions of the model lie on providing a multiobjective integrated water quantity and quality system approach for a number of hydro-climate and water demand components (rivers or channels, reservoirs and their hydraulic components, rural or urban water withdraws, irrigation, between others.) as well as their probable constraints, that are able to couple with a wider range of study applications. To achieve the required efficiency, appropriate linearization techniques and sequential linear programming were used. The implemented multiobjective function allows one to more rationally and efficiently provide studies of quali-quantitative water allocation according to its required priorities and water quality concentration levels. The developed tool is well suited to water resources planning and management and decision making.

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REFERENCES

[1] Azevedo, L. G. T., Porto, R. L. L., Porto, M. (1998), Sistema de apoio à decisão para o gerenciamento integrado de quantidade e qualidade da água: metodologia e estudo de caso. Revista Brasileira de Recursos Hídricos, Porto Alegre, v.3, n. 1, p. 21-51. DOI: 10.21168/rbrh.v3n1.p21-51

[2] Berhe, F.T., Melesse, A.M., Hailu, D., Sileshi, Y. (2013), MODSIM-based water allocation modeling of Awash River Basin, Ethiopia; Journal Catena, V. 109, p. 118–128. https://doi.org/10.1016/j.catena.2013.04.007

[3] Curi, R. C., Cucri, W. F., Oliveira, M. B. A. (2005), Análise de Alterações na Receita Líquida de um Perímetro Irrigado no Semiárido sob Condições de Variações Hídricas e Econômicas. Revista Brasileira de Recursos Hídricos. vol. 9, n. 3, p. 39-53. DOI: 10.21168/rbrh.v9n3.p39-53

[4] CONAMA (2005), Resolução Nº 357, Ministério do meio ambiente, www.mma.gov.br/conama - access in 15/09/2010.

[5] DHI (2001). Short Introduction and Tutorial, Water & Environment, MIKE BASIN, www.mikepoweredbydhi.com/products/mike-hydro-basin, acess in 05/01/2021.

[6] Huang, Y. (2014), Multi-objective calibration of a reservoir water quality model in aggregation and non-dominated sorting approaches. Journal of Hydrology, v.510 p. 280–292. https://doi.org/10.1016/j.jhydrol.2013.12.036

[7] Han, M., Qingwang, R., Wang, Y.; Du, J.; Hao, Z., Sun, F., Cheng, L., Qi, S., Li, D., (2013) Integrated Approach to Water Allocation in River Basins. Journal of Water Resources Planning and Management. v. 139. p.159-165. DOI: 10.1061/(ASCE)WR.1943-5452.0000255

[8] Hu, D., Yaob, S., Duana, C., Lia, S. (2020); Real-time simulation of hydrodynamic and scalar transport in large riverlake systems; Journal of Hydrology, Volume 582, 124531. https://doi.org/10.1016/j.jhydrol.2019.124531

[9] Hashimoto, T., Stedinger, J. R., Loucks, D. P. (1982), Reliability, Resiliency, and Vulnerability Criteria for Water Resource System Performance Evaluation. Water Resources Research, vol. 18, n. 1, p. 14-20. https://doi.org/10.1029/WR018i001p00014

[10] Kaiglová, J., Langhammer, J., (2013) Analysis of efficiency of pollution reduction measures in rural basin using MIKE BASIN model. Case study: Olšava River Basin. Journal of Hydrology and Hydromechanics, v. 62, p. 43–54. DOI: 10.2478/johh-2014-0007

[11] MATLAB (2010), Moler, Cleve, Reprinted from The MathWorks News & Notes, www.mathworks.com/products.html?s_tid=gn_ps, acess academic version in: 02/02/2010.

[12] Latorre, J. M., Cerisola, S., Ramos, A., Perea, A., Bellido, R. (2014), Coordinated Hydropower Plant Simulation for Multireservoir Systems. Journal of Water Resources Planning and Management, Vol. 140, nº. 2. DOI: 10.1061/(ASCE)WR.1943-5452.0000306.

[13] Lamparelli, M. C. (2004), Graus de trófia em corpos de água no Estado de São Paulo, 238, Doctoral Thesis, Institute of Geosciences, Federal University of São Paulo, acess in: www.researchgate.net/ - 18/02/2010.

[14] Labadie, J. W. (2004), Discussion of “Optimal Operation of Multireservoir Systems: State-of-the-Art Review”. Journal of Water Resources Planning and Management. Vol. 131 Issue 5, p406-407. 3p. DOI: 10.1061/(ASCE)0733-9496(2004)130:2(93)

[15] Lanna, A.E. (1998), Tópicos Avançados em Programação Linear: Método Simplex Revisado, Análise Pós-Ótimo e Artifícios de Linearização. Graduate Program in Water Resources and Sanitation at the Federal University of Rio Grande do Sul. p. 75-87, acess in: www.ufrgs.br/producao/ - 23/05/2009.

[16] Lee, S., Pomeroy, C., and Burian, S. (2021), Setting Future Water Rates for Sustainability of a Water Distribution System; Journal of Water Resources Planning and Management, New York. 147(2): 04020108. DOI: 10.1061/(ASCE)WR.1943-5452.0001313

[17] Loucks, D. P. (2000), Sustainable water resources management. Water International Resource Association, v. 25, n. 1 p. 3-10. https://doi.org/10.1080/02508060008686793

[18] Quintela, A.C. (1981), Hidráulica, Calouste Gulbenkian Foundation, Lisbon, Portugal.

[19] Ray, P. A., Kirshen, P. H., Vogel, R. M. (2010), Integrated Optimization of a Dual Quality Water and Wastewater System. Journal of Water Resources Planning and Management. v. 136. p.37-47. DOI: 10.1061/(ASCE)WR.1943-5452.0000004

[20] Roberto, A. N. (2002), Modelos de Rede de Fluxo Para Alocação da Água Entre Múltiplos Usos em uma Bacia Hidrográfica. Master's Dissertation, Polytechnic School of the University of São Paulo, São Paulo, 116 p.

[21] Simonovic, S. P. (1992), Reservoir system analysis: closing gap between theory and practice. of Water Resources Planning and Management, v. 118, nº 3, p. 262 – 280. ISSN Journal 0733-9496/92/0003- 0262

[22] Tucci, E. M. T. (2005), Modelos Hidrológicos, collaboration of the Brazilian water resources association/ABRH. 2ª edição. Publisher of the Federal University of Rio Grande do Sul, Porto Alegre, Brazil.

[23] Vieira, A. S., Curi, R. C., Curi, W. F., Santos, V. S (2012); Simulação, via Programação Linear Sequencial com Artifícios de Linearização, para Sistema de Recursos Hídricos Parte 1 e Parte 2: Modelo e Aplicação. Revista Brasileira de Recursos Hídricos, v.17 n.1, p.33-51. DOI: 10.21168/rbrh.v17n1.p33-51

[24] Villela, S.M. & Mattos, A. (1975), Hidrologia Aplicada, McGraw-Hill, São Paulo - SP.

[25] Von Sperling, M. (1995), Introdução à qualidade das águas e ao tratamento de esgotos. Department of Sanitary and Environmental Engineering; Federal University of Minas Gerais, V. 01, 2º ed., p. 243.

[26] Wurbs, R. A. (2005), Comparative Evaluation of Generalized River/Reservoir System Models. Technical Report n. 282. Texas Water Resources Institute, acess in: www.oaktrust.library.tamu.edu/handle/1969.1/6092 - 08/03/2009.

[27] Zhao, Q. & Cai, X. (2020), Deriving representative reservoir operation rules using a hidden Markov-decision tree model, Advances in Water Resources, 146, 103753. https://doi.org/10.1016/j.advwatres.2020.103753